SECTION- B

Part (i) :-

Solution :-

$= \int x e^{(5x^2+1)} \, dx$

Multiplying and dividing by 10

$= \frac{1}{10} \int 10x \, e^{(5x^2+1)} \, dx$ — (i)

let $5x^2+1 = t$

Diff w.r.t t

$\frac{dt}{dx} = \frac{d}{dx}(5x^2+1)$

$\frac{dt}{dx} = \frac{d}{dx}(5x^2) + \frac{d}{dx}(1)$

$\frac{dt}{dx} = 10x + 0$

$dt = 10x \, dx$ — (ii)

Put in eq (i)

$= \frac{1}{10} \int e^t \, dt$

$= \frac{1}{10} \cdot e^t + C$

put $t = 5x^2+1$

$\boxed{= \frac{1}{10} \cdot e^{5x^2+1} + C}$

Part (ii) :-

Solution :-

$y = \tan^{-1}\left(\frac{x}{p}\right)$

Diff w.r.t x

$\frac{dy}{dx} = \frac{d}{dx}\left[\tan^{-1}\left(\frac{x}{p}\right)\right]$

$\frac{dy}{dx} = \frac{1}{1+\left(\frac{x}{p}\right)^2} \cdot \frac{d}{dx}\left(\frac{x}{p}\right)$ $\because \frac{dy}{dx} \tan^{-1} = \frac{1}{1+x^2}$

$\frac{dy}{dx} = \frac{1}{1+x^2/p^2} \cdot \frac{1}{p} \cdot \frac{d}{dx}(x)$

$\frac{dy}{dx} = \frac{1}{\frac{p^2+x^2}{p^2}} \cdot \frac{1}{p} \cdot 1$

$\frac{dy}{dx} = \frac{p^2}{p^2+x^2} \cdot \frac{1}{p}$

$\boxed{\frac{dy}{dx} = \frac{p}{p^2+x^2}}$

Part (iii) :-

Solution :-

$= \lim_{x \to 5} \frac{\sqrt{x}-\sqrt{5}}{x-5} = \frac{\sqrt{5}-\sqrt{5}}{5-5} = \left(\frac{0}{0}\right)$ form

Conjugate :-

$= \lim_{x \to 5} \frac{\sqrt{x}-\sqrt{5}}{x-5} \times \frac{\sqrt{x}+\sqrt{5}}{\sqrt{x}+\sqrt{5}}$

$$= \lim_{x\to 5} \frac{(\sqrt{x})^2-(\sqrt{5})^2}{(x-5)(\sqrt{x}+\sqrt{5})}$$

$$= \lim_{x\to 5} \frac{(\cancel{x-5})}{(\cancel{x-5})(\sqrt{x}+\sqrt{5})}$$

$$= \lim_{x\to 5} \frac{1}{\sqrt{x}+\sqrt{5}}$$

Applying limit,

$$= \frac{1}{\sqrt{5}+\sqrt{5}} = \frac{1}{2\sqrt{5}}$$

$$\boxed{\text{Answer} = \frac{1}{2\sqrt{5}}}$$

Part (iv) :-

$f(x) = (\tan x)^2$

Diff w.r.t x

$\frac{d}{dx}(f(x)) = \frac{d}{dx}(\tan x)^2$

$f'(x) = 2\tan x \cdot \frac{d}{dx}(\tan x)$

$f'(x) = 2\tan x \sec^2 x \rightarrow$ ∵ $\sec^2\theta = 1+\tan^2\theta$

$f'(x) = 2\tan x(1+\tan^2 x)$

$f'(x) = 2\tan x + 2\tan^3 x$

Diff again w.r.t x

$\frac{d}{dx}(f'(x)) = \frac{d}{dx}(2\tan x + 2\tan^3 x)$

$f''(x) = 2\frac{d}{dx}\tan x + 2\frac{d}{dx}(\tan x)^3$

$f''(x) = 2\sec^2 x + 2\cdot 3\tan^2 x \cdot \frac{d}{dx}(\tan x)$

$f''(x) = 2\sec^2 x + 6\tan^2 x \sec^2 x$

$f''(x) = 2(1+\tan^2 x) + 6\tan^2 x(1+\tan^2 x)$

$f''(x) = 2 + 2\tan^2 x + 6\tan^2 x + 6\tan^4 x$

Diff again w.r.t x

$\frac{d}{dx}(f''(x)) = \frac{d}{dx}(2) + 2\frac{d}{dx}(\tan x)^2 + 6\frac{d}{dx}(\tan x)^2$

$+ 6\frac{d}{dx}(\tan x)^4$

04 + 1 = 5

$\rightarrow f'''(x) = 0 + 4\tan x \cdot \frac{d}{dx}(\tan x) + 12\tan x \cdot \frac{d}{dx}(\tan x)$

$+ 24\tan^3 x \cdot \frac{d}{dx}(\tan x)$

$\rightarrow f'''(x) = 4\tan x\sec^2 x + 12\tan x \sec^2 x$

$+ 24\tan^3 x \sec^2 x$

Answer ↑

Part (V) :-

Solution :-

$g(x) = |\sin x\hat{i} - 2x\hat{j} + \cos x\hat{k}|$

taking absolute

$g(x) = \sqrt{(\sin x)^2 + (-2x)^2 + (\cos x)^2}$

$g(x) = \sqrt{\sin^2 x + \cos^2 x + 4x^2}$

$g(x) = \sqrt{1+4x^2}$

diff w.r.t x

$\frac{d}{dx}[g(x)] = \frac{d}{dx}(1+4x^2)^{1/2}$

$g'(x) = \frac{1}{2}(1+4x^2)^{-1/2} \cdot \frac{d}{dx}(1+4x^2)$

$g'(x) = \frac{1}{\cancel{2}\sqrt{1+4x^2}} \cdot \cancel{8}^{4}x$

$$\boxed{g'(x) = \frac{4x}{\sqrt{1+4x^2}}}$$

Part (vi) :-

Solution :-

$y + 3x = 9$ —(i)

for - x-intercept,

Put $y = 0$ in (i)

$0 + 3x = 9$

$3x = 9$

$x = 9/3$

$x = 3$

For y-intercept,

Put $x = 0$ in eq (i)

$y + 3(0) = 9$

$y = 9$

The intercepts are,

$\boxed{\text{At } x,\ x = 3 \quad (3,0)}$ $\boxed{\text{At } y,\ y = 9 \quad (0,9)}$

Part (vii) :-

Solution :-

$x^2 + y^2 + 4x - 6y + 13 = 0$

Compare with

$x^2 + y^2 + 2gx + 2fx + c = 0$

Here :-

$2g = 4$ | $2f = -6$ | $c = 13$

$g = 2$ | $f = -3$ |

(i) We know that,

center $= (-g, -f)$

$\boxed{= (-2, 3)}$

ii) We know that,

$r = \sqrt{g^2+f^2-c}$

$r = \sqrt{(2)^2+(-3)^2-13}$

$r = \sqrt{4+9-13}$

$r = \sqrt{13-13}$

$r = \sqrt{0}$

$\boxed{r = 0}$ [As $\sqrt{g^2+f^2-c} = 0$, so circle is reduced to zero, point circle]

Part (viii):

Solution:

Focus = $(0,3)$

line of directrix = -3

Thus parabola is upward (+ve y).

focus = $(0,a)$

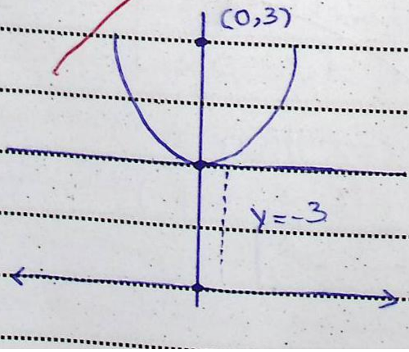
By comparing, $a=3$

Put $a=3$ in eq.

$x^2 = 4ay$

$x^2 = 4(3)y$

$\boxed{x^2 = 12y}$



Part (ix):

Solution:

$\frac{x^2}{4} + \frac{y^2}{9} = 1$

xing each term by 32

$32\left(\frac{x^2}{4}\right) + 32\left(\frac{y^2}{9}\right) = 32(1)$

$9x^2 + 4y^2 = 32$

Diff w.r.t x

$9\frac{d}{dx}x^2 + 4\frac{d}{dx}y^2 = \frac{d}{dx}32$

$18x + 8y\frac{dy}{dx} = 0$

$8y\frac{dy}{dx} = -18x$

$\frac{dy}{dx} = \frac{-18x}{8y}$

$\frac{dy}{dx} = \frac{-9x}{4y}$

$m = \frac{-9(x)}{4(y)}$

At point $(1,2)$

$m = \frac{-9(1)}{4(2)}$

$m = -9/8$

05

Equation of tangent : $(\underset{x_1}{1}, \underset{y_1}{2})$

$y - y_1 = m(x - x_1)$

$y - 2 = -\frac{9}{8}(x - 1)$

$y = -\frac{9}{8}x + \frac{9}{8} + 2$

$y = -\frac{9}{8}x + \frac{9+16}{8}$

$$\boxed{y = -\frac{9}{8}x + \frac{25}{8}}$$

Part (x) :-

$= \int_1^2 (2x^{-2} - 3)\,dx$

$= \int_1^2 2x^{-2} - 3\int_1^2 dx$

$= 2\frac{x^{-1}}{-1}\Big|_1^2 - 3x\Big|_1^2$

$= -2x^{-1}\Big|_1^2 - 3x\Big|_1^2$

$= -2(2^{-1} - 1^{-1}) - 3(2-1)$

$= -2\left(\frac{1}{2} - \frac{1}{1}\right) - 3$

$= -\cancel{2}(-\frac{1}{\cancel{2}}) - 3$

$= +1 - 3$ $\boxed{= -2}$

05

SECTION - C

LONG - QUESTIONS

→ Question No :- 6

Part (a)

$= \int_1^4 x^3\,dx$

$= \frac{x^4}{4}\Big|_1^4$

$= \frac{1}{4}\cdot x^4\Big|_1^4$

$= \frac{1}{4}(4^4 - 1^4)$

$= \frac{1}{4}(256 - 1)$

$$\boxed{= \frac{255}{4}}$$

05

Part (b)

$7x - 10y + 13 = 0$

In general form,

$10y = 7x + 13$

dividing each term by 10

$\frac{10y}{10} = \frac{7}{10}x + \frac{13}{10}$

$\boxed{y = \frac{7}{10}x + \frac{13}{10}}$ $\because y = mx + c$

Slope-Intercept form.

here, $m = \frac{7}{10}$, $c = \frac{13}{10}$

Question No :- 5

Part(a) :-

$f(x) = 2x + 7$ —(i), $g(x) = 2x$ —(ii)

(i) $f[g(x)]$:-

Put $x = g(x)$ in (i)

$f[g(x)] = 2[g(x)] + 7$

$f(g(x)) = 2(2x) + 7$ $\because g(x) = 2x$

$f(g(x)) = 4x + 7$

ii) $g(f(x))$

Put $x = f(x)$ in ii

$g(f(x)) = 2(f(x))$

$g(f(x)) = 2(2x + 7)$

$\boxed{g(f(x)) = 4x + 14}$

Part (b) :-

$y = x^2 \cdot e^{\sin x}$

diff w.r.t x

$\frac{dy}{dx} = \frac{d}{dx}(x^2 \cdot e^{\sin x})$

$\frac{dy}{dx} = x^2 \frac{d}{dx} e^{\sin x} + e^{\sin x} \cdot \frac{d}{dx} x^2$

$\frac{dy}{dx} = x^2 \cdot e^{\sin x} \cdot \frac{d}{dx}(\sin x) + e^{\sin x} \cdot (2x)$

$\frac{dy}{dx} = x^2 e^{\sin x} \cos x + 2x e^{\sin x}$

Question No 3 :-

$= \int \frac{1}{x^2 + 4} dx \longrightarrow \int \frac{1}{x^2 + (2)^2}$

let $x = 2\tan\theta$ —(i) $\because x = a\tan\theta$ for $x^2 + a^2$

diff w.r.t x

$$\frac{d(x)}{dx} = 2\frac{d}{dx}(\tan\theta)$$

$$1 = 2\sec^2\theta \cdot \frac{d\theta}{dx}$$

$$dx = 2\sec^2\theta \, d\theta$$

Put in question,

$$= \int \frac{2\sec^2\theta \, d\theta}{(2\tan\theta)^2 + (4)}$$

$$= \int \frac{2\sec^2\theta}{4\tan^2\theta + 4} d\theta$$

$$= \int \frac{\cancel{2}^1 \sec^2\theta}{\cancel{4}_2(\tan^2\theta + 1)} d\theta$$

$$= \frac{1}{2}\int \frac{\sec^2\theta}{\tan^2\theta + 1} d\theta$$

$$= \frac{1}{2}\int \frac{\sec^2\theta}{\sec^2\theta} d\theta \qquad \because \tan^2\theta + 1 = \sec^2\theta$$

$$= \frac{1}{2}\int (1) d\theta$$

$$= \frac{1}{2}\theta + C \quad \text{— ii}$$

Finding θ from (i)

$$x = 2\tan\theta$$

$$\theta = \tan^{-1}\left(\frac{x}{2}\right)$$

Put $\theta = \tan^{-1}\left(\frac{x}{2}\right)$ in (ii)

$$\boxed{= \frac{1}{2}\tan^{-1}(x/2) + C}$$

Part (b)

$l_1 = 7x + 3y - 6 = 0, \quad l_2 = 5x - 2y + 2 = 0$

Slope of line 'l_1' $= \frac{-\text{coeff of } x}{\text{co-eff of } y} = -\frac{7}{3}$

$$m_1 = -7/3$$

Slope of line 'l_2' $= \frac{-\text{co-eff of } x}{\text{co-eff of } y} = \frac{-5}{-2}$

$$m_2 = 5/2$$

We know that,

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\tan\theta = \frac{(-7/3) - 5/2}{1 + (5/2)(-7/3)}$$

$$\tan\theta = \left|\frac{-7/3 - 5/2}{1 - 35/6}\right| \longrightarrow \tan\theta = \frac{\frac{-14-15}{\cancel{6}}}{\frac{6-35}{\cancel{6}}}$$

$\tan\theta = \frac{+29}{+29}$

$\tan\theta = +1$

$\theta = \tan^{-1}(+1)$

$\boxed{\theta = 45^\circ}$ (acute)

and for obtuse $\angle = 180^\circ - 45^\circ = \boxed{135^\circ}$

The angle of intersection from l_1 to l_2 is 45°

$\longleftrightarrow$