بسم اللہ الرحمن الرحیم

(THE MOST MERCIFUL AND BENIFICIAL)

Q.NO 2

SECTION-B

PART —— (i)

$= \int x e^{(5x^2+1)} dx$ —— (i)

let $u = 5x^2+1$

Taking derivative on both sides w.r.t x

$\frac{du}{dx} = \frac{d}{dx} 5x^2+1$ put bracket

$\frac{du}{dx} = 10x$

$\frac{du}{10} = x\, dx$ —— (ii)

put (ii) in (i)

$= \int e^u \frac{du}{10}$

$= \frac{1}{10} \int e^u$

$= \frac{1}{10} e^u + C$ —— (iii)

Now replacing $u = 5x^2+1$ in (iii)

$= \boxed{\frac{1}{10} e^{5x^2+1} + C}$

PART —— (ii)

$$y = \tan^{-1}\left(\frac{x}{p}\right) \text{ —— (i)}$$

Now taking derivative of (i) w.r.t x

$$\frac{dy}{dx} = \frac{d(y)}{dx} \tan^{-1}\left(\frac{x}{p}\right)$$

$$\frac{dy}{dx} = \frac{1}{1+\left(\frac{x}{p}\right)^2} \frac{d}{dx}\frac{x}{p}$$

By ∵ $\boxed{\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}\frac{d}{dx}x}$

So $\frac{dy}{dx} = \frac{1}{\frac{1+x^2}{p^2}} \frac{1}{p}\frac{d}{dx}x$

$$\frac{dy}{dx} = \frac{1}{\frac{p^2+x^2}{p^2}} \cdot \frac{1}{p}$$

$$\frac{dy}{dx} = \frac{p^2}{p^2+x^2} \cdot \frac{1}{p}$$

$$\boxed{\frac{dy}{dx} = \frac{p}{p^2+x^2}}$$

PART —— (iii)

$$\lim_{x \to 5} \frac{\sqrt{x}-\sqrt{5}}{x-5} \text{ —— (i)}$$

Now taking rationalization of (i)

$$= \lim_{x \to 5} \frac{\sqrt{x}-\sqrt{5}}{x-5} \times \frac{\sqrt{x}+\sqrt{5}}{\sqrt{x}+\sqrt{5}}$$

$$= \lim_{x \to 5} \frac{(\sqrt{x})^2 - (\sqrt{5})^2}{(x-5)(\sqrt{x}+\sqrt{5})}$$

$$= \lim_{x \to 5} \frac{\cancel{(x-5)}}{\cancel{(x-5)}(\sqrt{x}+\sqrt{5})}$$

$$= \lim_{x \to 5} \frac{1}{\sqrt{x}+\sqrt{5}} \quad \text{—— (ii)}$$

Now put $x = 5$ in (ii)

$$= \frac{1}{\sqrt{5}+\sqrt{5}}$$

$$\boxed{= \frac{1}{2\sqrt{5}}}$$

05

PART —— (IV)

$$f(x) = \tan^2 x \quad \text{—— (i)}$$

We will take first derivative of (i) w.r.t x.

$$\frac{d}{dx} f(x) = \frac{d}{dx} \tan^2 x$$

$$f'(x) = 2 \tan x \frac{d}{dx} \tan x$$

0A

$$f'(x) = 2 \tan x \sec^2 x \quad \text{—— (ii)}$$

Now taking second derivative w.r.t x of (ii)

04

$$\frac{d}{dx} f'(x) = \frac{d}{dx} 2 \tan x \sec^2 x$$

$$f''(x) = 2\frac{d}{dx}\tan x\sec^2 x$$

Now applying product rule

$$= 2\left[\tan x\frac{d}{dx}\sec^2 x + \sec^2 x\frac{d}{dx}\tan x\right]$$

$$= 2\left[\tan x \cdot 2\sec x\frac{d}{dx}\sec x + \sec^2 x\sec^2 x\right]$$

$$= 2\left[\tan x\sec x\cdot\sec x\tan x + \sec^4 x\right]$$

$$f''(x) = 2\left[\tan^2 x\sec^2 x + \sec^4 x\right] \quad \text{(iii)}$$

Now we will take third derivative w.r.t x 7 (iii)

$$\frac{d}{dx}f''(x) = \frac{d}{dx}2\left[\tan^2 x\sec^2 x + \sec^4 x\right]$$

$$= 2\left(\frac{d}{dx}\tan^2 x\sec^2 x + \frac{d}{dx}\sec^4 x\right)$$

$$= 2\left(\left[\tan^2 x\frac{d}{dx}\sec^2 x + \sec^2 x\frac{d}{dx}\tan^2 x\right]\right.$$

missing

$$\left. + 4\sec^3 x\frac{d}{dx}\sec x\right)$$

$$= 2\left[\tan^2 x\cdot 2\sec x\frac{d}{dx}\sec x + \sec^2 x \cdot 2\tan x\frac{d}{dx}\tan x\right] + 4\sec^3 x\cdot\sec x\cdot\tan x$$

$$= 2\left(\left[\tan^2 x\cdot\sec x\cdot\tan x\cdot\sec x + \tan x\sec^2 x\right] + \sec^4 x\tan x\right)$$

$$= 2\left[\tan^3 x \cdot \sec^2 x + \tan x \sec^2 x + \sec^4 x \tan x\right]$$

Wrong Answer

$$\boxed{= 2\tan^3 x \cdot \sec^2 x + 2\tan x \sec^2 x + 2\sec^4 x \tan x}$$

PART —— (V)

$$g(x) = |\sin x\hat{i} - 2x\hat{j} + \cos x\hat{k}| \quad \text{—— (1)}$$

Now taking modulus of (1)

$$g(x) = \sqrt{(\sin x)^2 + (\cos x)^2 + (-2x)^2}$$

$$g(x) = \sqrt{\sin^2 x + \cos^2 x + 4x^2}$$

$\because \sin^2 x + \cos^2 x = 1$

$$g(x) = \sqrt{1 + 4x^2} \quad \text{—— (ii)}$$

Now taking derivative of (ii)

$$\frac{d}{dx} g(x) = \frac{d}{dx}(1+4x^2)^{1/2}$$

$$= \frac{1}{2}(1+4x^2)^{\frac{1}{2}-1} \frac{d}{dx} 1+4x^2$$

$$= \frac{1}{\cancel{2}}(1+4x^2)^{-1/2} \cdot \cancel{8}^{4} x$$

$$\boxed{= \frac{4x}{\sqrt{1+4x^2}}}$$

PART —— (Vi)

$$y + 3x = 9 \quad \text{—— (1)}$$

For x intercept $y = 0$

so put $y = 0$ in (1)

$$0 + 3x = 9 \Rightarrow x = 3$$

∴ (3,0) is x intercept.

Now

for y intercept $x=0$

$$y+3x=9$$

$$y+0=9$$

$$y=9$$

∴ (0,9) is y intercept

∴ $\boxed{(3,0)(0,9)}$ are required x and y intercepts.

PART —— (vii)

13

$$x^2+y^2+4x-6y+3=0$$

Comparing with

$$x^2+y^2+2gx+2fy+c=0$$

$$2gx=4x$$

$$g=2$$

$$2fy=-6y$$

$$f=-3$$

for (i) Centre $C(-g,-f)$

$$\boxed{=(-2,3)}$$

Now for Radius we have formula

$$r=\sqrt{g^2+f^2-c}$$

$$r=\sqrt{(2)^2+(-3)^2-13}$$

$$r=\sqrt{4+9-13}$$

$$r=\sqrt{13-13}$$

$$\boxed{r=0}$$

PART —— (xii)

$$\frac{dy}{dx} = \frac{1}{x^2}, \; y(2) = 0$$

$$\frac{dy}{dx} = \frac{1}{x^2}$$

$$dy = \frac{dx}{x^2} \text{ —— (i)}$$

Now integrating (i)

$$\int dy = \int \frac{dx}{x^2}$$

$$y = \int x^{-2}\, dx$$

$$y = \frac{x^{-1}}{-1}$$

$$y = -\frac{1}{x} + C \text{ —— (ii)}$$

Now when $y = 0$
$x = 2$ put these values in (ii)

$$0 = -\frac{1}{2} + C$$

$$= -\frac{1}{2} + C$$

$$C = \frac{1}{2}$$

put $C = \frac{1}{2}$ in (ii)

$$\boxed{y = -\frac{1}{x} + \frac{1}{2}}$$

PART —— (x)

$$\int_1^2 (2x^{-2} - 3)\, dx$$

Now integrating (i)

$$= \int_1^2 2x^{-2} dx + \int_1^2 -3\, dx$$

$$= 2\int_1^2 x^{-2}\, dx - 3\int_1^2 dx$$

$$= 2\left(\frac{x^{-1}}{-1} - 3x\right)\Big|_1^2$$

$$= -\frac{2}{x}\Big|_1^2 - 3x\Big|_1^2$$

Now finding definite integral

$$= -\frac{2}{x}\Big|_1^2 - 3x\Big|_1^2$$

$$= -2\left(\frac{1}{2} - 1\right) - 3(2 - 1)$$

$$= -\cancel{2}\left(\frac{-1}{\cancel{2}}\right) - 3(1)$$

$$= +1 - 3$$

$$\boxed{= -2}$$

PART —— (IX)

$$\frac{x^2}{4} + \frac{y^2}{9} = 1 \quad \text{at } (1, 2)$$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1 \quad \text{—— (1)}$$

Taking derivative of (1) w.r.t x.

e xfm

$$\frac{d}{dx}\frac{x^2}{4} + \frac{d}{dx}\frac{y^2}{9} = \frac{d}{dx}1$$

B

$$\frac{1}{4}\frac{d}{dx}x^2 + \frac{1}{9}\frac{d}{dx}y^2 = 0$$

$$\frac{1}{4}2x + \frac{1}{9}2y\frac{dy}{dx} = 0.$$

$$\frac{1}{2}x + \frac{2}{9}y\frac{dy}{dx} = 0$$

$$\frac{2}{9}y\frac{dy}{dx} = -\frac{1}{2}x.$$

$$\frac{dy}{dx} = -\frac{9}{4}\frac{x}{y}$$

Now for slope
put $P(1,2)$
in (ii)

$m = -\frac{9}{4}\left(\frac{1}{2}\right)$

$m = -\frac{9}{8}$

Now finding equation of tangent

$y - y_1 = m(x - x_1)$

$(y - 2) = -\frac{9}{8}(x - 1)$

$8y - 16 = -9x + 4$

$8y + 9x - 25 = 0$

Divide by 8 (iii)

$\boxed{y + \frac{9}{8}x - \frac{25}{8} = 0}$

PART —— (viii)

$F(0,3)$ directrix $y = -3$

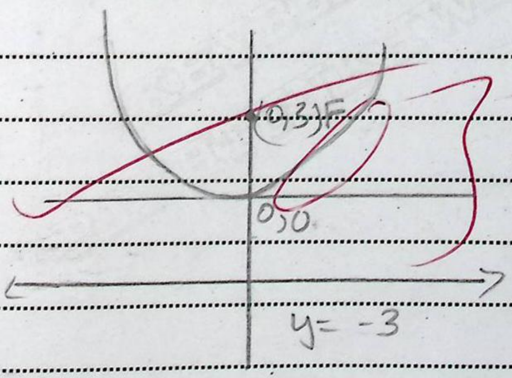
it means $a = 3$

So $x^2 = 4ay$

$x^2 = 4(3)y$

$\boxed{x^2 = 12y}$

SKETCH :-



PART — (xi)

$f(x,y) = (x^2+xy+y^2)^{-1}$ — (i)

let $f(x,y) = z$

Now put λ in both sides of (i)

$f(\lambda x, \lambda y) = ((\lambda x)^2 + (\lambda x)(\lambda y) + (\lambda y)^2)^{-1}$

$= (\lambda^2 x^2 + \lambda^2 xy + \lambda^2 y^2)^{-1}$

Extra:

$= (\lambda^2)^{-1} (x^2+xy+y^2)^{-1}$

$= \lambda^{-2} (x^2+xy+y^2)^{-1}$

so $n = -2$

Now taking partial derivative of (i) w.r.t x

$\frac{\partial}{\partial x} f(x,y) = \frac{\partial}{\partial x} (x^2+xy+y^2)^{-1}$

$= -(x^2+xy+y^2)^{-2} \frac{\partial}{\partial x} (x^2+xy+y^2)$

$= -(x^2+xy+y^2) \cdot 2x + y$ — (ii)

Now multiply (ii) with x.

$x \frac{\partial z}{\partial x} = -(x^2+xy+y^2) 2x^2 + xy$

Now taking partial derivative of (i) w.r.t y

$\frac{\partial}{\partial y} f(x,y) = \frac{\partial}{\partial y} (x^2+xy+y^2)^{-1}$

Extra

$$\frac{\partial}{\partial y} z = -1(x^2+xy+y^2)^{-2} \frac{\partial}{\partial y} x^2+xy+y^2$$

$$= -(x^2+xy+y^2)^{-2} \cdot 2y + x \text{ — (iii)}$$

Now multiply (iii) with y

Extra

$$= -(x^2+xy+y^2)^{-2} \cdot 2y^2 + xy$$

Now by Euler theorem

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nZ$$

$$-(x^2+xy+y^2)^{-2}\, 2x^2+xy + -(x^2+xy+y^2)^{-2}(2y^2+xy)$$

$$= -(x^2+xy+y^2)^{-2}(2x^2+2xy+2y^2)$$

$$= -(x^2+xy+y^2)^{-2}\, 2(x^2+xy+y^2)$$

$$= -2(x^2+xy+y^2)^{-1}$$

$$= -2f(x) = \boxed{nZ}.$$

SECTION-C

Q. NO 6

(a)

$$= \int_1^4 x^3\,dx \text{ — (i)}$$

Integrating (i).

$$= \left.\frac{x^4}{4}\right|_1^4$$

Now finding definite integral

$$= \frac{(4)^4}{4} - \frac{(1)^4}{4}$$

$= \frac{256}{4} - \frac{1}{4}$

$= \boxed{\frac{255}{4}}$

(b)

$7x - 10y + 13 = 0$

We will convert to slope intercept form which is

$y = mx + c$

$-10y = -7x - 13$

Divide by -10

$\frac{-10}{-10}y = \frac{-7}{-10}x - \frac{13}{-10}$

$\boxed{y = \frac{7}{10}x + \frac{13}{10}}$.

Q.NO 5

(a)

$f(x) = 2x + 7$ $g(x) = 2x$.

$f(g(x)) = 2(2x) + 7)$

$= \boxed{4x + 7}$

~~Now~~ finding

$g(f(x)) = 2(2x + 7)$

$= \boxed{4x + 14}$

(b)

$$y = x^2 \cdot e^{\sin x} \quad \text{(i)}$$

Now differentiating w.r.t x equation (i)

$$\frac{dy}{dx} = \frac{d}{dx} x^2 \cdot e^{\sin x}$$

Applying Product rule.

$$\frac{dy}{dx} = x^2 \frac{d}{dx} e^{\sin x} + e^{\sin x} \frac{d}{dx} x^2.$$

$$= x^2 e^{\sin x} \frac{d}{dx} \sin x + e^{\sin x} \cdot 2x$$

$$= \boxed{x^2 \cdot e^{\sin x} \cdot \cos x + 2x \cdot e^{\sin x}}$$

OR

$$x \cdot e^{\sin x} \left[x \cos x + 2 \right]$$

Q.NO 3

(a)

$$= \int \frac{dx}{x^2+4}$$

$$= \int \frac{dx}{x^2+(2)^2}$$

so $a = 2$

let $x = a \tan\theta$

$x = 2 \tan\theta$

Taking derivative of (ii) w.r.t θ

$$\frac{dx}{d\theta} = 2 \frac{d}{d\theta} \tan\theta$$

$$\frac{dx}{d\theta} = 2\sec^2\theta$$

$$dx = 2\sec^2\theta \, d\theta$$

put (iii) in (i)

$$= \int \frac{2\sec^2\theta \, d\theta}{(2\tan\theta)^2 + (2)^2}$$

$$= \int \frac{2\sec^2\theta \, d\theta}{4\tan^2\theta + 4}$$

$$= \int \frac{\cancel{2}\sec^2\theta \, d\theta}{\cancel{4}_2(1+\tan^2\theta)}$$

$$= \frac{1}{2}\int \frac{\sec^2\theta \, d\theta}{\sec^2\theta} \qquad \because 1+\tan^2\theta = \sec^2\theta$$

$$= \frac{1}{2}\int d\theta$$

$$= \frac{1}{2}\theta$$

from (ii) equation (iv) becomes

$$= \boxed{\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)} + C$$

(b)

$L_1 = 7x+3y-6 \qquad L_2 = 5x-2y+2$

$3y = -7x+6 \qquad -2y = -5x-2$

$y = -\frac{7}{3} + \frac{6}{3} \qquad y = +\frac{5}{2}x + 1$

$\text{slope} = -\frac{7}{3} \qquad m_2 = \frac{5}{2}$

$$\tan\theta = \frac{m_1 - m_2}{1+m_1m_2}$$

$$= \frac{-\frac{7}{3} - \frac{5}{2}}{1 - \left(\frac{7}{3}\right)\frac{5}{2}}$$

$$= \frac{-\frac{29}{6}}{1 - \frac{35}{6}} \implies$$

$$= -\cancel{\frac{2a}{b}} \div -\frac{2a}{b}$$

$$= -\frac{2a}{b} \times \frac{b}{-2a}$$

$$\tan\theta = 1$$

$$\theta = \tan^{-1}(1)$$

$$\boxed{= 45^\circ}$$

5