(SECTION-B)

(QUESTION-2)

(PART-i)

$\int xe^{(5x^2+1)}dx$

Let

$5x^2+1 = t \quad -i$

Diff w.r.t x

$\frac{d(5x^2+1)}{dx} = \frac{dt}{dx}$

$5\frac{d(x^2)}{dx} + \frac{d(1)}{dx} = \frac{dt}{dx}$

$5(2x) + 0 = \frac{dt}{dx}$

$10x = \frac{dt}{dx}$

$xdx = \frac{dt}{10} \quad -ii$

put i and ii in question:

$$= \int e^t \frac{dt}{10}$$

$$= \frac{1}{10} \int e^t dt$$

$$= \frac{1}{10} \cdot \frac{e^t}{\frac{d}{dt}(t)} + c$$

$$= \frac{1}{10} \cdot \frac{e^t}{1} + c$$

$$= \frac{1}{10} e^t + c \Rightarrow \boxed{\frac{1}{20} e^{(5x^2+1)} + c} \text{ Ans.}$$

(Part - ii)

$$y = \tan^{-1}\left(\frac{x}{p}\right)$$

Diff w.r.t x $\qquad \because \frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$

$$\frac{dy}{dx} = \frac{d}{dx} \tan^{-1}\left(\frac{x}{p}\right)$$

$$= \frac{1}{1+\left(\frac{x}{p}\right)^2} \cdot \frac{d}{dx}\left(\frac{x}{p}\right)$$

$$= \frac{1}{1+\frac{x^2}{p^2}} \cdot \left(\frac{1}{p}\right)$$

$$= \frac{1}{\frac{p^2+x^2}{p^2}} \cdot \frac{1}{p}$$

$$= \frac{p^2}{p^2+x^2} \cdot \frac{1}{p}$$

$$\boxed{= \frac{p}{p^2+x^2}} \text{ Ans.}$$

(Part iii)

$$\lim_{x \to 5} \frac{\sqrt{x} - \sqrt{5}}{x-5}$$

$$= \lim_{x \to 5} \frac{\sqrt{x} - \sqrt{5}}{x-5}$$

÷ing & ×ing $\sqrt{x} + \sqrt{5}$

$$= \lim_{x \to 5} \frac{\sqrt{x} - \sqrt{5}}{x-5} \times \frac{\sqrt{x} + \sqrt{5}}{\sqrt{x} + \sqrt{5}}$$

$$= \lim_{x \to 5} \frac{(\sqrt{x})^2 - (\sqrt{5})^2}{(x-5)(\sqrt{x}+\sqrt{5})}$$

$$= \lim_{x \to 5} \frac{(x-5)}{(x-5)(\sqrt{x}+\sqrt{5})}$$

$$= \lim_{x \to 5} \frac{1}{\sqrt{x}+\sqrt{5}}$$

applying the limit

$$= \frac{1}{\sqrt{5}+\sqrt{5}}$$

$$\boxed{= \frac{1}{2\sqrt{5}}}$$ Ans.

(PART Vi)

$y + 3x = 9$ x-intercept=? y-intercept=?

for x-intercept:-

put $y=0$

$0 + 3x = 9$

$3x = 9$

$x = 3$

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For y-intercept: put $x=0$

$y + 3(0) = 9$

$y = 9$

x-intercept => $(3, 0)$

y-intercept => $(0, 9)$

(PART Vii)

$x^2 + y^2 + 4x - 6y + 13 = 0$ - i

Compare eq(i) with general eq of circle

$x^2 + y^2 + 2gx + 2fy + c$

$2g = 4$	$2f = -6$	$c = 13$
$g = 2$	$f = -3$	

Centre = $(-g, -f) = (-2, 3)$

For radius:-

$r = \sqrt{g^2 + f^2 - c}$

$r = \sqrt{(2)^2 + (-3)^2 - 13}$

$r = \sqrt{4 + 9 - 13}$

$r = \sqrt{13 - 13}$

$r = 0$

Centre = $(-2, 3)$, radius = 0

(Part - viii)

As directrix $= -3$ & Focus $= (0,3)$ – i

So The equation of Parabola for positive y-axis is $(x-h)^2 = 4a(y-k)$ – iii

from the focus we can see that

$h = 0 \quad , k = 0$

for a we know that

Focus $= F = (0, a)$ – ii

compare i and ii

$a = 3$

Now put $h=0, k=0, a=3$ in iii

$(x-0)^2 = 4(3)(y-0)$

$x^2 = 12(y)$

$x^2 = 12y$

(Part - X)

$$\int_1^2 (2x^{-2} - 3)dx$$

$$= \int_1^2 \left(\frac{2}{x^2} - 3\right)dx$$

$$= \int_1^2 \frac{2}{x^2} \cancel{dx} - \int_1^2 3dx$$

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$$= 2\int_1^2 \frac{1}{x^2}dx - 3\int_1^2 dx$$

$$= 2\int_1^2 x^{-2}dx - 3x\Big|_1^2$$

$$= 2\frac{x^{-2+1}}{-2+1}\Big|_1^2 - 3x\Big|_1^2$$

$$= -2x^{-1}\Big|_1^2 \cancel{- 3x\Big|_1^2}$$

$$= \left[\frac{-2}{x} - 3x\right]\Big|_1^2$$

$$= \left[\frac{-2}{x} - 3x\right] - \cancel{\left[\frac{-2}{x} - 3x\right]}\Big|_1^2$$

$$= \left[\frac{-2}{2} - 3(2)\right] - \left[\frac{-2}{1} - 3(1)\right]$$

$= [-1-6-(-2-3)]$

$= [-7-(-5)]$

$= [-7+5]$

$= -2+C$ Ans

(Part-Xi)

$f(x,y) = f = (x^2+xy+y^2)^{-1}$

put

$x = \lambda x$ and $y = \lambda y$

$f(\lambda x,\lambda y) = ((\lambda x)^2 + (\lambda x)(\lambda y) + (\lambda y)^2)^{-1}$

$= (\lambda^2 x^2 + \lambda^2 xy + \lambda^2 y^2)^{-1}$

$= (\lambda^2)^{-1}(x^2+xy+y^2)^{-1}$

$= \lambda^{-2} f(x,y)$

it is a homogeneous function of degree -2

By Euler's theorm:-

$$x\frac{df}{dx} + y\frac{df}{dy} = -2f$$

$f = (x^2+xy+y^2)^{-1}$

Diff w.r.t x

$$\frac{df}{dx} = \frac{d}{dx}(x^2+xy+y^2)^{-1}$$

$$\frac{df}{dx} = -1(x^2+xy+y^2)^{-1-1} \cdot \frac{d}{dx}(x^2+xy+y^2)$$

$$= -(x^2+xy+y^2)^{-2}(2x+y)$$

$$\frac{df}{dx} = \frac{-(2x+y)}{(x^2+xy+y^2)^2}$$

xing "x" on both sides

$$x\frac{df}{dx} = \frac{-x(2x+y)}{(x^2+xy+y^2)^2}$$

$$x\frac{df}{dx} = \frac{-2x^2-xy}{(x^2+xy+y^2)^2}$$

$f = (x^2+xy+y^2)^{-1}$

Diff w.r.t y

$$\frac{df}{dy} = \frac{d}{dy}(x^2+xy+y^2)^{-1}$$

$$\frac{df}{dy} = -1\,(x^2+xy+y^2)^{-1-1} \cdot \frac{d}{dy}(x^2+xy+y^2)$$

$$= -(x^2+xy+y^2)^{-2}\,(\ x+2y)$$

$$\frac{df}{dy} = -\frac{(x+2y)}{(x^2+xy+y^2)^2}$$

xing "y" on b.S

$$y\frac{df}{dy} = \frac{-y(x+2y)}{(x^2+xy+y^2)^2}$$

$$y\frac{df}{dy} = \frac{-yx-2y^2}{(x^2+xy+y^2)^2}$$

Now

By Euler's theorm

$$x\frac{df}{dx} + y\frac{df}{dy} = nz$$

$$x\frac{df}{dx} + y\frac{df}{dy} = \frac{-2x^2-xy}{(x^2+xy+y^2)} + \frac{(-yx-2y^2)}{(x^2+xy+y^2)}$$

$$= \frac{-2x^2-xy-xy-2y^2}{(x^2+xy+y^2)^2}$$

$$= \frac{-2x^2-2xy-2y^2}{(x^2+xy+y^2)^2}$$

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$$= \frac{-2(x^2+xy+y^2)}{(x^2+xy+y^2)^2}$$

$$= -2(x^2+xy+y^2)^{-1}$$

$$x\frac{df}{dx} + y\frac{df}{dy} = -2f$$ Hence proved.

(PART - XII)

$$\frac{dy}{dx} = \frac{1}{x^2}, \quad y(2) = 0$$

$$\frac{dy}{dx} = \frac{1}{x^2}$$

$$dy = \frac{1}{x^2}\,dx$$

Taking $\int$ on b.S

$$\int dy = \int \frac{1}{x^2}\,dx$$

$$y = \int x^{-2}\,dx$$

$$y = \frac{x^{-2+1}}{-2+1} + c$$

$$y = \frac{x^{-1}}{-1} + c$$

$$y = -x^{-1} + c$$

$$y = -\frac{1}{x} + c \quad \text{— i}$$

For c

$$y = -\frac{1}{x} + c$$

$$y + \frac{1}{x} = c$$

put $y=0$ and $x=2$

$$0 + \frac{1}{2} = c$$

$$c = \frac{1}{2} \quad \text{— ii}$$

put ii in i

$$y = -\frac{1}{x} + \frac{1}{2} \quad \text{Ans.}$$

(PART-5)

$$g(x) = |\sin x\,\hat{i} - 2x\,\hat{j} + \cos x\,\hat{k}|$$

As we know that

$$|a+b| = \sqrt{a^2+b^2}$$

then

$$|\sin x\,\hat{i} - 2x\,\hat{j} + \cos x\,\hat{k}| = \sqrt{(\sin x)^2 + (-2x)^2 + (\cos x)^2}$$

$$= \sqrt{\sin^2 x + 4x^2 + \cos^2 x}$$

$$= \sqrt{\sin^2 x + \cos^2 x + 4x^2}$$

$\because \cos^2 x + \sin^2 x = 1$

$$= \sqrt{1 + 4x^2}$$

$$g(x) = (1+4x^2)^{1/2}$$

Now Diff w.r.t x

$$\frac{d}{dx} g(x) = \frac{d}{dx}(1+4x^2)^{1/2}$$

$$g'(x) = \frac{1}{2}(1+4x^2)^{\frac{1}{2}-1} \cdot \frac{d}{dx}(1+4x^2)$$

$$= \frac{1}{2}(1+4x^2)^{-1/2}\,(\overset{4}{8x})$$

$$= \frac{4x}{(1+4x^2)^{1/2}}$$

$$g'(x) = \frac{4x}{\sqrt{1+4x^2}} \text{ Ans.}$$

EXTRA QUESTION:-

(PART - ix)

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

Diff w.r.t x

$$\frac{d}{dx}\left(\frac{x^2}{4}\right) + \frac{d}{dx}\left(\frac{y^2}{9}\right) = \frac{d}{dx}(1)$$

$$\frac{1}{4}\frac{d}{dx}(x^2) + \frac{1}{9}\frac{d}{dx}(y^2) = 0$$

$$\frac{1}{4_2}(2x) + \frac{1}{9}(2y)\frac{dy}{dx} = 0$$

$$\frac{1}{2}x + \frac{2y}{9}\frac{dy}{dx} = 0$$

$$\frac{2y}{9}\frac{dy}{dx} = -\frac{1}{2}x$$

$$\frac{dy}{dx} = -\frac{9x}{4y}$$

put x=1 and y=2

$$m = -\frac{9(1)}{4(2)}$$

$$\text{slope} = m = -\frac{9}{8}$$

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Equ OF TANGENT:-

$$y - y_1 = m(x - x_1) \quad \text{--- i}$$

put the following values in i

$$y_1 = 2, \; x_1 = 1, \; m = -\frac{9}{8}$$

$$y - 2 = -\frac{9}{8}(x-1)$$

$$8(y-2) = -9(x-1)$$

$$8y - 16 = -9x + 9$$

$$9x + 8y - 16 - 9 = 0$$

$$9x + 8y - 25 = 0$$

(SECTION- C)

(QUESTION-3)

(PART - A)

$$= \int \frac{dx}{x^2+4}$$

$$= \int \frac{dx}{(x)^2+(2)^2} \quad -i$$

So we know that

$x = a\tan Q$

here $x = x$ $a = 2$

$x = 2\tan Q$ -ii

Diff w.r.t x

$$\frac{dx}{dx} = 2\frac{d}{dx}\tan Q$$

$$1 = 2\sec^2 Q \frac{dQ}{dx}$$

$$dx = 2\sec^2 Q dQ \quad -iii$$

$$= \int \frac{2\sec^2 Q dQ}{(2\tan Q)^2+(2)^2}$$

$$= \int \frac{2\sec^2 Q dQ}{4\tan^2 Q+4}$$

$$= \int \frac{2\sec^2 Q dQ}{4(\tan^2 Q+1)}$$

$\because$ $\tan^2 Q+1 = \sec^2 Q$

$$= \frac{1}{2}\int \frac{\cancel{\sec^2 Q}dQ}{\cancel{\sec^2 Q}}$$

$$= \frac{1}{2}\int dQ$$

$$= \frac{1}{2}Q + c \quad -iv$$

Now from eq ii

$x = 2\tan Q$

$\frac{x}{2} = \tan Q$

$Q = \tan^{-1}\left(\frac{x}{2}\right) \quad -v$

put eq v in iv

$$\int \frac{dx}{x^2+4} = \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+c$$

(QUESTION-3)

(PART-B)

Angle between line L_1 $7x+3y-6=0$

and line L_2 $5x-2y+2=0$

First of all we will find slopes of both the lines.

$L_1 \Rightarrow 7x+3y-6=0$

then

$$m_1 = -\frac{\text{coeff of } x}{\text{coeff of } y}$$

$$m_1 = -\frac{7}{3}$$

$L_2 \Rightarrow 5x-2y+2=0$

then

$$m_2 = -\frac{\text{coeff of } x}{\text{coeff of } y}$$

$$m_2 = -\frac{5}{-2}$$

$$m_2 = \frac{5}{2}$$

To find angle from slopes we know that

$$\tan\theta = \frac{m_1 - m_2}{1+m_2m_1}$$

$$\tan\theta = \frac{-\frac{7}{3} - \left(\frac{5}{2}\right)}{1+\left(\frac{5}{2}\right)\left(-\frac{7}{3}\right)}$$

$$= \frac{-\frac{7}{3} - \frac{5}{2}}{1 - \frac{35}{6}}$$

$$= \frac{\frac{-14-15}{6}}{\frac{6-35}{6}}$$

$$= \frac{+29}{+29}$$

$$\tan\theta = 1$$

$\tan\theta = 1$

$\theta = \tan^{-1}(1)$

$\theta = 45°$

(QUESTION-5)

(PART A)

$f(x) = 2x+7 \quad g(x) = 2x$

$f(g(x)) = f(2x)$

Now

$f(2x) = 2(2x)+7$

$f(2x) = 4x+7$

$g(f(x)) = g(2x+7)$

$g(2x+7) = 2(2x+7)$

$= 4x+14$

$f(g(x)) = 4x+7$

$g(f(x)) = 4x+14$

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(QUESTION-5)

(PART-B)

$y = x^2 \cdot e^{\sin x}$

Diff w.r.t x

$\frac{dy}{dx} = \frac{d}{dx}(x^2 \cdot e^{\sin x})$

$= x^2 \frac{d}{dx} e^{\sin x} + e^{\sin x} \frac{d}{dx} x^2$

$= x^2 e^{\sin x} \frac{d}{dx}(\sin x) + e^{\sin x}(2x)$

$= x^2 e^{\sin x}(\cos x) + 2xe^{\sin x}$

$= x^2 e^{\sin x} \cos x + 2x e^{\sin x}$

$\frac{dy}{dx} = xe^{\sin x}(x \cos x + 2)$

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R.W

$y = 2x + 2$

$2x + 4(0) + 6 = 0$

$2x = -6$

$x = -3$

$\frac{d}{dx} f(x) = 2xy + y^2$

$\sqrt[3]{1+25+1}$

$= \sqrt[3]{27}$

$= 3$

$6x^2 +$

$\frac{(x-\cancel{2})(x+2)}{(\cancel{x-2})}$

$= 2+2 = 4$

$= \frac{x^{2+1}}{2+1} \Big|_1^4$

$= \frac{x^3}{3}$

$= \frac{1}{3}\left(x^3 - x^3\right)\Big|_1^4$

$= \frac{1}{3}(64-1)$

$= \frac{63}{3} = 21$

(QUESTION-6)

(PART-A)

$= \int_1^4 x^3 dx$

$= \frac{x^{3+1}}{3+1} \Big|_1^4 + c$

$= \frac{x^4}{4} \Big|_1^4 + c$

$= \frac{x^4}{4} + \cancel{c} - \frac{x^4}{4} - \cancel{c} \Big|_1^4 \Rightarrow \frac{1}{4}\left[x^4 - x^4\right]\Big|_1^4$

$= \frac{1}{4}\left[(4)^4 - (1)^4\right]$

$= \frac{1}{4}\left[256 - 1\right]$

$\int_1^4 x^3 dx = \frac{255}{4}$

(PART- B)

$7x - 10y + 13 = 0$

SLOPE - INTERCEPT FORM

$y = mx + c$

So the above eq will b

$7x - 10y + 13 = 0$

$7x + 13 = 10y$

$y = \frac{7}{10}x + \frac{13}{10}$

$m = \frac{7}{10}$, $c = \frac{13}{10}$