SECTION - B

QUESTION . 2 :

Part (i)

SOLUTION:

Let $5x^2+1 = t$

Diff w.r.t x:

$$10x + 0 = \frac{dt}{dx}$$

$$\frac{dt}{dx} = 10x$$

$$\frac{dt}{10} = x\,dx$$

Now, $= \int x e^{(5x^2+1)} dx$

$= \int e^t \frac{dt}{10}$

$= \frac{1}{10} \int e^t dt$

$= \frac{1}{10} \frac{e^t}{\frac{d}{dt}(t)} + c$

$= \frac{1}{10} \cdot \frac{e^t}{1} + c$

$= \frac{1}{10} e^t + c$

Put $t = 5x^2+1$

$$\boxed{= \frac{1}{10} e^{5x^2+1} + c}$$

Part (ii)

SOLUTION:

$$y = \tan^{-1}\left(\frac{x}{p}\right)$$

Diff w.r.t x:

$\frac{d}{dx} y = \frac{d}{dx} \tan^{-1}\left(\frac{x}{p}\right)$

$\frac{dy}{dx} = \frac{1}{1+\left(\frac{x}{p}\right)^2} \cdot \frac{d}{dx}\left(\frac{x}{p}\right)$

$\frac{dy}{dx} = \frac{1}{1+\frac{x^2}{p^2}} \cdot \frac{1}{p} \frac{d}{dx} x$

$\frac{dy}{dx} = \frac{1}{\frac{p^2+x^2}{p^2}} \cdot \frac{1}{p} \cdot 1$

$\frac{dy}{dx} = \frac{p^{\cancel{2}}}{p^2+x^2} \cdot \frac{1}{\cancel{p}}$

$$\boxed{\frac{dy}{dx} = \frac{p}{p^2+x^2}}$$

Part (iii)

SOLUTION:

$$= \lim_{x \to 5} \frac{\sqrt{x} - \sqrt{5}}{x - 5} = \frac{\sqrt{5} - \sqrt{5}}{5 - 5} = \frac{0}{0} \text{ form}$$

× and ÷ by conjugate:

$$= \lim_{x \to 5} \frac{\sqrt{x} - \sqrt{5}}{x - 5} \cdot \frac{\sqrt{x} + \sqrt{5}}{\sqrt{x} + \sqrt{5}}$$

$$= \lim_{x \to 5} \frac{(\sqrt{x} - \sqrt{5})(\sqrt{x} + \sqrt{5})}{(x-5)(\sqrt{x} + \sqrt{5})}$$

$$= \lim_{x \to 5} \frac{(\sqrt{x})^2 - (\sqrt{5})^2}{(x-5)(\sqrt{x} + \sqrt{5})}$$

$$= \lim_{x \to 5} \frac{\cancel{x-5}}{\cancel{(x-5)}(\sqrt{x} + \sqrt{5})}$$

$$= \lim_{x \to 5} \frac{1}{\sqrt{x} + \sqrt{5}}$$

Applying the limit:

$$= \frac{1}{\sqrt{5} + \sqrt{5}}$$

$$\boxed{= \frac{1}{2\sqrt{5}}}$$

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Part (v):

SOLUTION:

$$g(x) = |\sin x\,\hat{i} - 2x\,\hat{j} + \cos x\,\hat{k}|$$

$$g(x) = \sqrt{\sin^2 x + 4x^2 + \cos^2 x}$$

$$g(x) = \sqrt{\sin^2 x + \cos^2 x + 4x^2}$$

$$g(x) = \sqrt{1 + 4x^2} \quad (\because \sin^2 x + \cos^2 x = 1)$$

Now, differentiate w.r.t x:

$$\frac{d}{dx} g(x) = \frac{d}{dx} (1 + 4x^2)^{\frac{1}{2}}$$

$$g'(x) = \frac{1}{2} (1 + 4x^2)^{\frac{1}{2} - 1} \cdot \frac{d}{dx} (1 + 4x^2)$$

$$g'(x) = \frac{1}{2}(1+4x^2)^{-\frac{1}{2}} \cdot (0+8x)$$

$$g'(x) = \frac{1}{\cancel{2}} \cdot \frac{1}{(1+4x^2)^{\frac{1}{2}}} (\overset{4}{\cancel{8}}x)$$

$$\boxed{g'(x) = \frac{4x}{\sqrt{1+4x^2}}}$$

Part (vi)

SOLUTION:

For x-intercept, put $y=0$

$$y+3x = 9$$
$$0+3x = 9$$
$$3x = 9$$
$$x = \frac{9}{3}$$
$$x = 3$$

$$\therefore \boxed{x\text{-intercept} = (3, 0)}$$

For y-intercept, put $x=0$

$$y+3x = 9$$
$$y+3(0) = 9$$
$$y+0 = 9$$
$$y = 9$$

$$\therefore \boxed{y\text{-intercept} = (0, 9)}$$

Part (vii)

SOLUTION:

$$x^2+y^2+4x-6y+13 = 0$$

Compare with.

$$x^2+y^2+2gx+2fy+c = 0$$

$$\Rightarrow \quad 2g = 4 \; ; \; 2f = -6 \; ; \; c = 13$$
$$g = 2 \qquad f = -3$$

As we know,

$$\text{Center} = (-g, -f)$$

$$\boxed{\text{Center} = (-2, 3)}$$

Now, radius $= \sqrt{g^2+f^2-c}$

$= \sqrt{(2)^2+(-3)^2-13}$

$= \sqrt{4+9-13}$

$= \sqrt{13-13}$

$\boxed{r = 0}$

Part (viii)

SOLUTION:

Focus $= (0,3)$

Compare with:

Focus $= (0,a)$

$\Rightarrow \quad a = 3$

As directrix $y=-3$

So the parabola is on positive y-axis.

For y-axis, equation of parabola is:

$x^2 = 4ay$ —— (i)

Put $a = 3$ in (i):

$x^2 = 4(3)y$

$\boxed{x^2 = 12y}$

Part (ix)

SOLUTION:

$$\frac{y^2}{9} + \frac{x^2}{4} = 1 \text{ —— (i)}$$

Compare with,

$$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$$ (As ellipse is on y-axis)

$\Rightarrow \quad a^2 = 9 \quad ; \quad b^2 = 4$

$a = 3 \quad ; \quad b = 2$

We know that,

$y - y_1 = m(x - x_1)$

For slope:

Diff (i) w.r.t x:

$$\frac{d}{dx}\frac{y^2}{9}+\frac{d}{dx}\frac{x^2}{4}=\frac{d}{dx}(1)$$

$$\frac{1}{9}\frac{d}{dx}y^2+\frac{1}{4}\frac{d}{dx}x^2=0$$

$$\frac{1}{9}(2y)\frac{dy}{dx}+\frac{1}{\cancel{4}^{2}}(\cancel{2}x)=0$$

$$\frac{2y}{9}\frac{dy}{dx}+\frac{x}{2}=0$$

$$\frac{2y}{9}\frac{dy}{dx}=-\frac{x}{2}$$

$$\frac{dy}{dx}=-\frac{x}{2}\cdot\frac{9}{2y}$$

$$\frac{dy}{dx}=-\frac{9x}{4y}$$

Put $x=1$, $y=2$

$$m=-\frac{9(1)}{4(2)}$$

$$m=-\frac{9}{8}$$

Put $m=-\frac{9}{8}$ in equation of tangent:

$$y-y_1=m(x-x_1)$$

$$y-2=-\frac{9}{8}(x-1)$$

$$8(y-2)=-9(x-1)$$

$$8y-16=-9x+9$$

$$9x+8y-16-9=0$$

$$\boxed{9x+8y-25=0}$$

Part (xi)

SOLUTION:

$$f(x,y)=z=(x^2+xy+y^2)^{-1} \text{ —— (i)}$$

Checking if it is homogenous:

Put $x=\lambda x$, $y=\lambda y$ in (i):

$$z=\left[(\lambda x)^2+(\lambda x)(\lambda y)+(\lambda y)^2\right]^{-1}$$

$$z = \frac{1}{\lambda^2 x^2 + \lambda^2 xy + \lambda^2 y^2}$$

$$z = \frac{1}{\lambda^2}\left[\frac{1}{x^2+xy+y^2}\right]$$

$$z = \lambda^{-2}(x^2+xy+y^2)^{-1}$$

So it is homogenous & $n=-2$

We know that Euler's theorem is:

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz$$

Differentiate (i) w.r.t x:

$$\frac{\partial}{\partial x} z = \frac{\partial}{\partial x}(x^2+xy+y^2)^{-1}$$

$$\frac{\partial z}{\partial x} = -1(x^2+xy+y^2)^{-1-1}\frac{\partial}{\partial x}(x^2+xy+y^2)$$

$$\frac{\partial z}{\partial x} = -(x^2+xy+y^2)^{-2}\left[\frac{\partial}{\partial x}x^2+\frac{\partial}{\partial x}xy+\frac{\partial}{\partial x}y^2\right]$$

$$\frac{\partial z}{\partial x} = -\frac{1}{(x^2+xy+y^2)^2}\left(2x+y\frac{\partial}{\partial x}x+0\right)$$

$$\frac{\partial z}{\partial x} = -\frac{1}{(x^2+xy+y^2)^2}(2x+y)$$

$$\frac{\partial z}{\partial x} = -\frac{2x+y}{(x^2+xy+y^2)^2}$$

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Differentiate (i) w.r.t y:

$$\frac{\partial}{\partial y} z = \frac{\partial}{\partial y}(x^2+xy+y^2)^{-1}$$

$$\frac{\partial z}{\partial y} = -1(x^2+xy+y^2)^{-1-1}\frac{\partial}{\partial y}(x^2+xy+y^2)$$

$$\frac{\partial z}{\partial y} = -1(x^2+xy+y^2)^{-2}\left[\frac{\partial}{\partial y}x^2+\frac{\partial}{\partial y}xy+\frac{\partial}{\partial y}y^2\right]$$

$$\frac{\partial z}{\partial y} = -\frac{1}{(x^2+xy+y^2)^2}\left(0+x\frac{\partial}{\partial y}y+2y\right)$$

$$\frac{\partial z}{\partial y} = -\frac{1}{(x^2+xy+y^2)^2}(x+2y)$$

$$\frac{\partial z}{\partial y} = -\frac{x+2y}{(x^2+xy+y^2)^2}$$

Put in Euler's theorem:

$$= x\left[-\frac{2x+y}{(x^2+xy+y^2)^2}\right] + y\left[-\frac{x+2y}{(x^2+xy+y^2)^2}\right]$$

$$= -\frac{x(2x+y)}{(x^2+xy+y^2)^2} - \frac{y(x+2y)}{(x^2+xy+y^2)^2}$$

$$= -\frac{2x^2+xy}{(x^2+xy+y^2)^2} - \frac{xy+2y^2}{(x^2+xy+y^2)^2}$$

$$= \frac{-(2x^2+xy)-(xy+2y^2)}{(x^2+xy+y^2)^2}$$

$$= \frac{-2x^2-xy-xy-2y^2}{(x^2+xy+y^2)^2}$$

$$= \frac{-2x^2-2xy-2y^2}{(x^2+xy+y^2)^2}$$

$$= -2\left[\frac{\cancel{x^2+xy+y^2}}{(x^2+xy+y^2)^{\cancel{2}}}\right]$$

$$= -2\left(\frac{1}{x^2+xy+y^2}\right)$$

$$= -2(x^2+xy+y^2)^{-1}$$

$$= -2z$$

Hence proved

Part (xii)

SOLUTION:

$$\frac{dy}{dx} = \frac{1}{x^2} \quad ; \quad y(2) = 0$$

$x = 2$

$y = 0$

$$dy = \frac{1}{x^2}\,dx$$

Taking $\int$ on b.s:

$$\int dy = \int \frac{1}{x^2}\,dx$$

$$y = \int x^{-2}\,dx$$

$$y = \frac{x^{-2+1}}{-2+1} + c$$

$$y = \frac{x^{-1}}{-1} + c$$

$$y = -\frac{1}{x} + c \quad \text{(General Solution)}$$

Put $x = 2$, $y = 0$

$$0 = -\frac{1}{2} + c$$

$$c = \frac{1}{2}$$

$$\therefore \boxed{y = -\frac{1}{x} + \frac{1}{2}}$$ (Particular solution)

SECTION - C

QUESTION . 3

Part (a)

SOLUTION :

$$= \int \frac{1}{x^2+4}\, dx$$

Compare with $\frac{1}{x^2+a^2}$:

$$\Rightarrow \quad a = 2$$

As we know that,

$$x = a \tan\theta \quad \text{—— (ii)}$$

Here $x = x$, $a = 2$

$$x = 2\tan\theta$$

Diff w.r.t x :

$$\frac{dx}{dt} = 2\frac{d}{dx}\tan\theta$$

$$1 = 2\sec^2\theta \frac{d\theta}{dx}$$

$$dx = 2\sec^2\theta\, d\theta \quad \text{—— (iii)}$$

$$= \int \frac{2\sec^2\theta\, d\theta}{4\tan^2\theta + 4}$$

$$= \int \frac{2\sec^2\theta\, d\theta}{4(\tan^2\theta + 1)}$$

$$= \int \frac{\cancel{2}^{1}\cancel{\sec^2\theta}\, d\theta}{{}^{2}\cancel{4}\,\cancel{\sec^2\theta}}$$

$$= \frac{1}{2}\int d\theta$$

$$= \frac{1}{2}\theta + c \quad \text{—— (iv)}$$

Now from eq. (ii)

$$x = a \tan \theta$$

$$x = 2 \tan \theta$$

$$\frac{x}{2} = \tan \theta$$

$$\theta = \tan^{-1}\left(\frac{x}{2}\right) \text{ ——— (v)}$$

Put (v) in (iv)

$$\boxed{\int \frac{dx}{x^2+4} = \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + c}$$

Part (b)

SOLUTION:

$L_1 : \quad 7x + 3y - 6 = 0$

$L_2 : \quad 5x - 2y + 2 = 0$

$$m_1 = \frac{-\text{coeff of } x}{\text{coeff of } y} = -\frac{7}{3}$$

$$m_2 = \frac{-\text{coeff of } x}{\text{coeff of } y} = \frac{+5}{+2} = \frac{5}{2}$$

Now,

$$\tan \theta = \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\tan \theta = \frac{-\frac{7}{3} - \frac{5}{2}}{1 + \left(-\frac{7}{3}\right)\left(\frac{5}{2}\right)}$$

$$\tan \theta = \frac{\frac{-14-15}{6}}{1 - \frac{35}{6}}$$

$$\tan \theta = \frac{-29}{6} \div \frac{6-35}{6}$$

$$\tan \theta = \frac{-29}{6} \div \frac{-29}{6}$$

$$\tan\theta = \frac{29}{6}x \frac{6}{29}$$

$$\tan\theta = 1$$

$$\theta = \tan^{-1}(1)$$

$$\boxed{\theta = 45^\circ}$$

Question . 5:

Part (a)

Solution:

$$f(x) = 2x+7$$

$$g(x) = 2x$$

$$f(g(x)) = 2(2x)+7$$

$$= 4x+7$$

$$g(f(x)) = 2(2x+7)$$

$$= 4x+14$$

Part (b)

Solution:

$$y = x^2 \cdot e^{\sin x}$$

Diff w.r.t x:

$$\frac{dy}{dx} = x^2 \frac{d}{dx} e^{\sin x} + e^{\sin x} \frac{d}{dx} x^2$$

$$\frac{dy}{dx} = x^2 \cdot e^{\sin x} \frac{d}{dx} \sin x + e^{\sin x}(2x)$$

$$\boxed{\frac{dy}{dx} = x^2 e^{\sin x} \cos x + 2x e^{\sin x}}$$

Fic. No. AUS

(صرف بورڈ کے استعمال کیلئے) اُمیدوار یہاں کچھ نہ لکھیں

QUESTION. 6:

Part (a)

SOLUTION:

$$= \int_1^4 x^3\,dx$$

$$= \frac{x^{3+1}}{3+1} + c \Big|_1^4$$

$$= \frac{x^4}{4} + c \Big|_1^4$$

Applying the limit:

$$= \left(\frac{(4)^4}{4} + c\right) - \left(\frac{(1)^4}{4} + c\right)$$

$$= \left(\frac{256}{4} + c\right) - \left(\frac{1}{4} + c\right)$$

$$= \frac{256}{4} + \cancel{c} - \frac{1}{4} - \cancel{c}$$

$$= \frac{256-1}{4}$$

$$\boxed{= \frac{255}{4}}$$

Part (b)

SOLUTION:

$$7x - 10y + 13 = 0$$

$$-10y = -7x - 13$$

$$\cancel{-}10y = \cancel{-}(7x+13)$$

$$10y = 7x + 13$$

$$\boxed{y = \frac{7}{10}x + \frac{13}{10}}$$ (slope-intercept ~~form~~)

$$\Rightarrow \quad m = \frac{7}{10}, \quad c = \frac{13}{10}$$